

## FINAL STATE INTERACTIONS IN $\varepsilon'/\varepsilon$

Stefano Bertolini

*INFN and SISSA, Via Beirut 4, I-34013 Trieste, Italy*

*E-mail: bertolin@he.sissa.it*

### Abstract

I shortly review the present status of the theoretical estimates of  $\varepsilon'/\varepsilon$ . I consider a few aspects of the theoretical calculations which may be relevant in understanding the present experimental results. In particular, I discuss the role of final state interactions and in general of non-factorizable contributions for the explanation of the  $\Delta I = 1/2$  selection rule in kaon decays and  $\varepsilon'/\varepsilon$ . Lacking reliable lattice calculations, the  $1/N$  expansion and phenomenological approaches may help in understanding correlations among theoretical effects and the experimental data. The same dynamics which underlies the CP conserving selection rule may drive  $\varepsilon'/\varepsilon$  in the range of the recent experimental measurements.

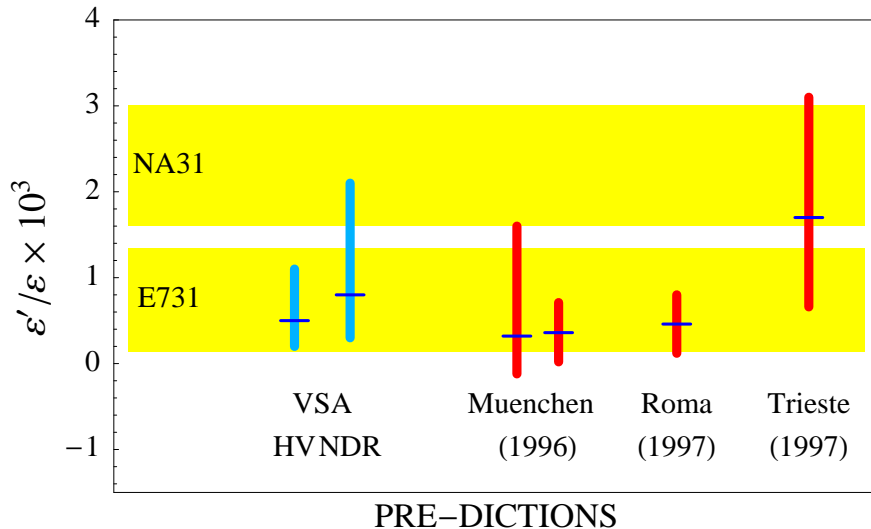


Figure 1: The 1- $\sigma$  results of the NA31 and E731 Collaborations (early 90's) are shown by the gray horizontal bands. The old München, Roma and Trieste theoretical predictions for  $\epsilon'/\epsilon$  are depicted by the vertical bars with their central values. For comparison, the VSA estimate is shown using two renormalization schemes.

The results announced during the last year by the KTeV <sup>1)</sup> and NA48 <sup>2)</sup> collaborations have marked a great experimental achievement, establishing 35 years after the discovery of CP violation in the neutral kaon system <sup>3)</sup> the existence of a much smaller violation acting directly in the decays.

While the Standard Model (SM) of strong and electroweak interactions provides an economical and elegant understanding of indirect ( $\epsilon$ ) and direct ( $\epsilon'$ ) CP violation in term of a single phase, the detailed calculation of the size of these effects implies mastering strong interactions at a scale where perturbative methods break down. In addition, CP violation in  $K \rightarrow \pi\pi$  decays is the result of a destructive interference between two sets of contributions, which may inflate up to an order of magnitude the uncertainties on the hadronic matrix elements of the effective four-quark operators. All that makes predicting  $\epsilon'/\epsilon$  a complex and subtle theoretical challenge <sup>4)</sup>.

The status of the theoretical predictions and experimental data available before the KTeV announcement in February 1999 is summarized in Fig. 1.

The gray horizontal bands show the one-sigma average of the old (early 90's) NA31 (CERN) and E731 (Fermilab) results. The vertical lines show the ranges of the published theoretical *predictions* (before February 1999), identified with the cities where most of the group members reside. The range of the naive Vacuum Saturation Approximation (VSA) is shown for comparison.

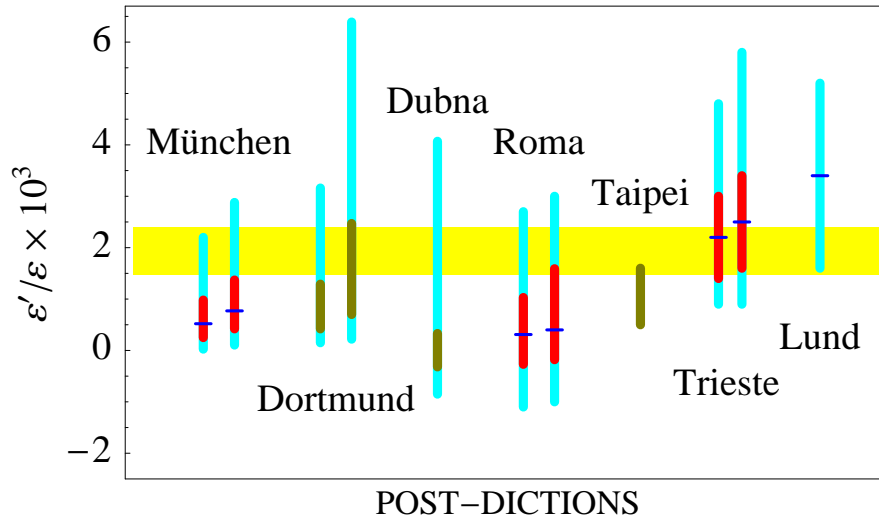


Figure 2: The latest theoretical calculations of  $\varepsilon'/\varepsilon$  are compared with the combined 1- $\sigma$  average of the NA31, E731, KTeV and NA48 results ( $\varepsilon'/\varepsilon = 19.2 \pm 4.6 \times 10^{-4}$ ), depicted by the gray horizontal band (the error is inflated according to the Particle Data Group procedure when averaging over data with substantially different central values).

The experimental and theoretical scenarios have changed substantially after the first KTeV data and the subsequent NA48 results. Fig. 2 shows the present experimental world average for  $\varepsilon'/\varepsilon$  compared with the revised or new theoretical calculations that appeared during the last year.

Notwithstanding the complexity of the problem, all theoretical calculations show a remarkable overall agreement, most of them pointing to a non-vanishing positive effect in the SM (which is by itself far from trivial).

On the other hand, if we focus our attention on the central values, many of the predictions prefer the  $10^{-4}$  regime, whereas only a few of them stand above  $10^{-3}$ . Is this just a “noise” in the theoretical calculations?

The answer is no. Without entering the details of the various estimates, it is possible to explain most of the abovementioned difference in terms of a single effect: the different size of the hadronic matrix element of the gluonic penguin  $Q_6$  obtained in the various approaches.

While some of the calculations, as the early München and Rome predictions, assume for  $\langle \pi\pi | Q_6 | K \rangle$  values in the neighborhood of the leading  $1/N$  result (naive factorization), other approaches, among which the Trieste and Dortmund calculations, find a substantial enhancement of this matrix element with respect to the simple factorization result. The bulk of such an effect is actually a global en-

hancement of the  $I = 0$  components of the  $K \rightarrow \pi\pi$  amplitudes, which affects *both* current-current *and* penguin operators, and it can be clearly understood in terms of chiral dynamics (final-state interactions).

As a matter of fact, one should in general expect an enhancement of  $\varepsilon'/\varepsilon$  with respect to the naive VSA due to final-state interactions (FSI). As Fermi first argued <sup>5)</sup>, in potential scattering the isospin  $I = 0$  two-body states feel an attractive interaction, of a sign opposite to that of the  $I = 2$  components thus affecting the size of the corresponding amplitudes. This feature is at the root of the enhancement of the  $I = 0$  amplitude over the  $I = 2$  one and of the corresponding enhancement of  $\varepsilon'/\varepsilon$  beyond factorization.

The question is how to make of a qualitative statement a quantitative one. Recent dispersive studies of the  $K \rightarrow \pi\pi$  amplitudes <sup>6)</sup> make use of the Omnès-Mushkelishvili representation <sup>7)</sup>

$$M(s + i\epsilon) = P(s) \exp \left( \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{\delta(s')}{s' - s - i\epsilon} ds' \right) , \quad (1)$$

in order to resum FSI effects from the knowledge of the  $\pi\pi$  rescattering phase  $\delta(s)$  in the elastic regime ( $s < 1 \text{ GeV}^2$ ).  $P(s)$  is a polynomial function of  $s$  which is related to the factorized amplitude. A solution of the above dispersive relation for the  $A_{0,2}$  amplitudes can be written as

$$A_I(s) = A' (s - m_\pi^2) \mathcal{R}_I(s) e^{i\delta_I(s)} , \quad (2)$$

where  $A'$  is the derivative of the amplitude at the subtraction point  $s = m_\pi^2$ . The coefficient  $\mathcal{R}$  represents the rescaling effect related to the FSI. By replacing  $A'_I$  with the value given by LO chiral perturbation theory, Pich and Pallante found  $\mathcal{R}(m_k^2)_{0,2} \simeq 1.4, 0.9$  thus confirming via a (partial) resummation of FSI the enhancement of the  $I = 0$  amplitudes, together with a mild depletion of the  $I = 2$  components.

The numerical significance of these results has been questioned <sup>8)</sup> on the basis that the precise size of the effect depends on the boundary conditions of the factorized amplitude which are not unambiguously known, due to the neglect of higher order chiral corrections. On the other hand, one should keep in mind that the choice of a low subtraction scale minimizes the effect of chiral corrections and makes the result of the dispersive analysis trustworthy.

At any rate a question remains: is the FSI rescaling on the factorized amplitudes, albeit a crucial contribution to the calculation of  $\varepsilon'/\varepsilon$ , all what we were looking for ?

Given the possibility that common systematic uncertainties may affect the calculation of  $\varepsilon'/\varepsilon$  and the  $\Delta I = 1/2$  rule (see for instance the present difficulties in calculating on the lattice “penguin contractions” for CP violating as well as for CP conserving amplitudes <sup>9)</sup>) a convincing calculation of  $\varepsilon'/\varepsilon$  must involve at the same time a reliable explanation of the  $\Delta I = 1/2$  selection rule, which is still missing. FSI effects alone are *not* enough to account for the large ratio of the  $I = 0, 2$  CP conserving amplitudes. Other sources of large non-factorizable corrections are needed, which may affect the determination of  $\varepsilon'/\varepsilon$  as well.

The  $\Delta I = 1/2$  selection rule in  $K \rightarrow \pi\pi$  decays is known since 45 years <sup>10)</sup> and it states the experimental evidence that kaons are 400 times more likely to decay in the  $I = 0$  two-pion state than in the  $I = 2$  component. This rule is not justified by any general symmetry consideration and, although it is common understanding that its explanation must be rooted in the dynamics of strong interactions, there is no up to date derivation of this effect from first principle QCD.

The ratio of  $I = 2$  over  $I = 0$  amplitudes appears directly in the definition of  $\varepsilon'/\varepsilon$  :

$$\frac{\varepsilon'}{\varepsilon} = \frac{1}{\sqrt{2}} \left\{ \frac{\langle (\pi\pi)_{I=2} | \mathcal{H}_W | K_L \rangle}{\langle (\pi\pi)_{I=0} | \mathcal{H}_W | K_L \rangle} - \frac{\langle (\pi\pi)_{I=2} | \mathcal{H}_W | K_S \rangle}{\langle (\pi\pi)_{I=0} | \mathcal{H}_W | K_S \rangle} \right\} . \quad (3)$$

As a consequence, a consistent calculation of  $\varepsilon'/\varepsilon$  must also address the determination of the CP conserving amplitudes.

The way we approach the calculation of the hadronic  $K \rightarrow \pi\pi$  transitions in gauge theories is provided by the Operator Product Expansion which allows us to write the relevant amplitudes in terms of the hadronic matrix elements of effective  $\Delta S = 1$  four quark operators (at a scale  $\mu$ ) and of the corresponding Wilson coefficients, which encode the information about the dynamical degrees of freedom heavier than the chosen renormalization scale:

$$\mathcal{H}_{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i \left[ z_i(\mu) + \tau y_i(\mu) \right] Q_i(\mu) . \quad (4)$$

The entries  $V_{ij}$  of the  $3 \times 3$  Cabibbo-Kobayashi-Maskawa matrix describe the flavour mixing in the SM and  $\tau = -V_{td}V_{ts}^*/V_{ud}V_{us}^*$ . For  $\mu < m_c$  ( $q = u, d, s$ ), the relevant

quark operators are:

$$\begin{aligned}
Q_1 &= (\bar{s}_\alpha u_\beta)_{V-A} (\bar{u}_\beta d_\alpha)_{V-A} \\
Q_2 &= (\bar{s}u)_{V-A} (\bar{u}d)_{V-A} \quad \left. \vphantom{\begin{aligned} Q_1 \\ Q_2 \end{aligned}} \right\} \text{Current-Current} \\
Q_{3,5} &= (\bar{s}d)_{V-A} \sum_q (\bar{q}q)_{V\mp A} \\
Q_{4,6} &= (\bar{s}_\alpha d_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V\mp A} \quad \left. \vphantom{\begin{aligned} Q_{3,5} \\ Q_{4,6} \end{aligned}} \right\} \text{Gluon "penguins"} \\
Q_{7,9} &= \frac{3}{2} (\bar{s}d)_{V-A} \sum_q \hat{e}_q (\bar{q}q)_{V\pm A} \\
Q_{8,10} &= \frac{3}{2} (\bar{s}_\alpha d_\beta)_{V-A} \sum_q \hat{e}_q (\bar{q}_\beta q_\alpha)_{V\pm A} \quad \left. \vphantom{\begin{aligned} Q_{7,9} \\ Q_{8,10} \end{aligned}} \right\} \text{Electroweak "penguins"}
\end{aligned} \tag{5}$$

Current-current operators are induced by tree-level W-exchange whereas the so-called penguin (and “box”) diagrams are generated via an electroweak loop. Only the latter “feel” all three quark families via the virtual quark exchange and are therefore sensitive to the weak CP phase. Current-current operators control instead the CP conserving transitions. This fact suggests already that the connection between  $\varepsilon'/\varepsilon$  and the  $\Delta I = 1/2$  rule is by no means a straightforward one.

Using the effective  $\Delta S = 1$  quark Hamiltonian we can write  $\varepsilon'/\varepsilon$  as

$$\frac{\varepsilon'}{\varepsilon} = e^{i\phi} \frac{G_F \omega}{2|\epsilon| \text{Re } A_0} \text{Im } \lambda_t \left[ \Pi_0 - \frac{1}{\omega} \Pi_2 \right] \tag{6}$$

where

$$\begin{aligned}
\Pi_0 &= \frac{1}{\cos \delta_0} \sum_i y_i \text{Re} \langle Q_i \rangle_0 (1 - \Omega_{\eta+\eta'}) \\
\Pi_2 &= \frac{1}{\cos \delta_2} \sum_i y_i \text{Re} \langle Q_i \rangle_2 \quad ,
\end{aligned} \tag{7}$$

and  $\langle Q_i \rangle \equiv \langle \pi\pi | Q_i | K \rangle$ . The rescattering phases  $\delta_{0,2}$  can be extracted from elastic  $\pi - \pi$  scattering data <sup>11)</sup> and are such that  $\cos \delta_0 \simeq 0.8$  and  $\cos \delta_2 \simeq 1$ . Given that the phase of  $\varepsilon$ ,  $\theta_\varepsilon$ , is approximately  $\pi/4$ , as well as  $\delta_0 - \delta_2$ ,  $\phi = \frac{\pi}{2} + \delta_2 - \delta_0 - \theta_\varepsilon$  turns out to be consistent with zero.

Two key ingredients appear in eq. 6:

1. The isospin breaking  $\pi^0 - \eta - \eta'$  mixing, parametrized by  $\Omega_{\eta+\eta'}$ , which is estimated to give a *positive* correction to the  $A_2$  amplitude of about 15-35%. However, it has been recently emphasized <sup>12)</sup> that NLO chiral corrections may make  $\Omega_{\eta+\eta'}$  negative and as large as  $-0.7$ , thus potentially providing a strong enhancement mechanism for  $\varepsilon'/\varepsilon$ . Further isospin breaking effects have been very recently discussed <sup>13)</sup>.
2. The combination of CKM elements  $\text{Im } \lambda_t \equiv \text{Im}(V_{ts}^* V_{td})$ , which affects directly the size of  $\varepsilon'/\varepsilon$  and the range of the uncertainty. The determination of  $\text{Im } \lambda_t$

goes through B-physics constraints and  $\varepsilon$ . The latter observable depends on the theoretical determination of  $B_K$ , the  $\bar{K}^0 - K^0$  hadronic parameter, which should be consistently determined within every approach. The theoretical uncertainty on  $B_K$  affects substantially the final uncertainty on  $\text{Im } \lambda_t$ . A better determination of the unitarity triangle is expected from the B-factories and the hadronic colliders <sup>14)</sup>. From K-physics,  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  gives the cleanest “theoretical” determination of  $\text{Im } \lambda_t$ , albeit representing a great experimental challenge <sup>15)</sup>.

A satisfactory approach to the calculation of  $\varepsilon'/\varepsilon$  should comply with the following requirements:

- A: A consistent definition of renormalized operators leading to the correct scheme and scale matching with the short-distance perturbative analysis.
- B: A self-contained calculation of *all* relevant hadronic matrix elements (including  $B_K$ ).
- C: A simultaneous explanation of the  $\Delta I = 1/2$  selection rule and  $\varepsilon'/\varepsilon$ .

None of the available calculations satisfies all previous requirements. I summarize very briefly the various attempts to calculate  $\varepsilon'/\varepsilon$  which have appeared so far and have lead to the estimates shown in Figs. 1 and 2. A simple naive approach to the problem is the VSA, which is based on two drastic assumptions: the factorization of the four quark operators and the vacuum saturation of the completeness of the intermediate states. As an example:

$$\begin{aligned} \langle \pi^+ \pi^- | Q_6 | K^0 \rangle &= 2 \langle \pi^- | \bar{u} \gamma_5 d | 0 \rangle \langle \pi^+ | \bar{s} u | K^0 \rangle - 2 \langle \pi^+ \pi^- | \bar{d} d | 0 \rangle \langle 0 | \bar{s} \gamma_5 d | K^0 \rangle \\ &\quad + 2 \left[ \langle 0 | \bar{s} s | 0 \rangle - \langle 0 | \bar{d} d | 0 \rangle \right] \langle \pi^+ \pi^- | \bar{s} \gamma_5 d | K^0 \rangle \end{aligned} \quad (8)$$

The VSA does not allow for a consistent matching of the scale and scheme dependence of the Wilson coefficients (the HV and NDR results are shown in Fig. 1) and it carries a potentially large systematic uncertainty <sup>4)</sup>. It is best used for LO estimates.

Generalized Factorization represents an attempt to address the issue of scale and renormalization scheme dependence in the framework of factorization by defining effective Wilson coefficients which absorb the perturbative QCD running of the quark operators. The new coefficients, scale and scheme independent at a given order in the strong coupling expansion, are matched to the factorized matrix

elements at the scale  $\mu_F$  which is arbitrarily chosen in the perturbative regime. A residual scale dependence remains in the penguin matrix elements via the quark mass. Fitting the  $\Delta I = 1/2$  rule and  $\varepsilon'/\varepsilon$  requires non-factorizable contributions both in the current-current and the penguin matrix elements, parametrized by (independent) phenomenological parameters <sup>16)</sup>.

In the München approach (phenomenological  $1/N$ ) some of the matrix elements are obtained by fitting the  $\Delta I = 1/2$  rule at  $\mu = m_c = 1.3$  GeV. On the other hand, the relevant gluonic and electroweak penguin  $\langle Q_6 \rangle$  and  $\langle Q_8 \rangle_2$  remain undetermined and are taken around their leading  $1/N$  values (which implies a scheme dependent result). In Fig. 2 the HV (left) and NDR (right) results are shown <sup>17)</sup>. The dark range represents the result of gaussian treatment of the input parameters compared to flat scanning (complete range).

In the recent years the Dortmund group has revived and improved the approach of Bardeen, Buras and Gerard <sup>18)</sup> based on the  $1/N$  expansion. Chiral loops are regularized via a cutoff and the amplitudes are arranged in a  $p^{2n}/N$  expansion. A particular attention has been given to the matching procedure between the scale dependence of the chiral loops and that arising from the short-distance analysis <sup>19)</sup>. The renormalization scheme dependence remains and it is included in the final uncertainty. The  $\Delta I = 1/2$  rule is reproduced, but the presence of the quadratic cutoff induces a matching scale instability (which is very large for  $B_K$ ). The NLO corrections to  $\langle Q_6 \rangle$  induce a substantial enhancement of the matrix element (right range in Fig. 2) compared to the leading order result (left). The dark range is drawn for central values of  $m_s$ ,  $\Omega_{\eta+\eta'}$ ,  $\text{Im } \lambda_t$  and  $\Lambda_{QCD}$ .

In the Nambu, Jona-Lasinio (NJL) modelling of QCD <sup>20)</sup> the Dubna group <sup>21)</sup> has calculated  $\varepsilon'/\varepsilon$  including chiral loops up to  $O(p^6)$  and the effects of scalar, vector and axial-vector resonances. Chiral loops are regularized via the heat-kernel method, which leaves unsolved the problem of the renormalization scheme dependence. A phenomenological fit of the  $\Delta I = 1/2$  rule implies deviations up to a factor two on the calculated  $\langle Q_6 \rangle$ . The reduced (dark) range in Fig. 2 corresponds to taking the central values of the NLO chiral couplings and varying the short-distance parameters.

In the approach of the Trieste group, based on the Chiral Quark Model ( $\chi$ QM) <sup>26)</sup>, all hadronic matrix elements are computed up to  $O(p^4)$  in the chiral expansion in terms of the three model parameters: the constituent quark mass, the quark condensate and the gluon condensate. These parameters are phenomenologically fixed by fitting the  $\Delta I = 1/2$  rule <sup>27)</sup>. This step is crucial in order to make the model predictive, since there is no a-priori argument for the consistency



of the matching procedure. As a matter of fact, all computed observables turn out to be very weakly dependent on the scale (and the renormalization scheme) in a few hundred MeV range around the matching scale, which is taken at 0.8 GeV as a compromise between the ranges of validity of perturbation theory and chiral expansion. The  $I = 0$  matrix elements are strongly enhanced by non-factorizable contributions and drive  $\varepsilon'/\varepsilon$  in the  $10^{-3}$  regime. The dark (light) ranges in Fig. 2 correspond to Gaussian (flat) scan of the input parameters. The bars on the left represent the results of ref. <sup>29)</sup> which updates the 1997 calculation <sup>28)</sup>. That on the right are a new estimate <sup>30)</sup>, similarly based on the  $\chi$ QM hadronic matrix elements, in which however  $\varepsilon'/\varepsilon$  is estimated in a novel way by including the explicit computation of  $\varepsilon$  in the ratio as opposed to the usual procedure of taking its value from the experiments. This approach has the advantage of being independent from the determination of the CKM parameters  $\text{Im } \lambda_t$  and of showing more directly the dependence on the long-distance parameter  $\hat{B}_K$  as determined within the model.

Lattice regularization of QCD is *the* consistent approach to the problem. On the other hand, there are presently important numerical and theoretical limitations, like the quenching approximation and chiral symmetry, which may substantially affect the calculation of the weak matrix elements. In addition, chiral perturbation theory is needed in order to obtain  $K \rightarrow \pi\pi$  amplitudes from the computed  $K \rightarrow \pi$  transitions. As summarized by Ciuchini at this conference <sup>9)</sup> lattice cannot provide us at present with reliable calculations of the  $I = 0$  penguin operators relevant to  $\varepsilon'/\varepsilon$ , as well as of the  $I = 0$  components of the hadronic matrix elements of the current-current operators (penguin contractions), which are relevant to the  $\Delta I = 1/2$  rule. This is due to large renormalization uncertainties, partly related to the breaking of chiral symmetry on the lattice. In this respect, very promising is the Domain Wall Fermion approach <sup>31)</sup> which allows us to decouple the chiral symmetry from the continuum limit <sup>32)</sup>. In the recent Roma re-evaluation of  $\varepsilon'/\varepsilon$   $\langle Q_6 \rangle$  is taken at the VSA value with a 100% uncertainty <sup>33)</sup>. The result is therefore scheme dependent (the HV and NDR results are shown in Fig. 2). The dark (light) ranges correspond to Gaussian (flat) scan of the input parameters.

The  $\Delta I = 1/2$  rule and  $B_K$  have been studied in the NJL framework and  $1/N$  expansion by Bijmans and Prades <sup>22)</sup> showing an impressive scale stability when including vector and axial-vector resonances. The same authors have recently produced a calculation of  $\varepsilon'/\varepsilon$  at the NLO in  $1/N$  <sup>23)</sup>. The calculation is done in the chiral limit and it is eventually corrected by estimating the largest  $SU(3)$  breaking effects. Particular attention is devoted to the matching between long- and short-distance components by use of the  $X$ -boson method <sup>24, 25)</sup>. The couplings of

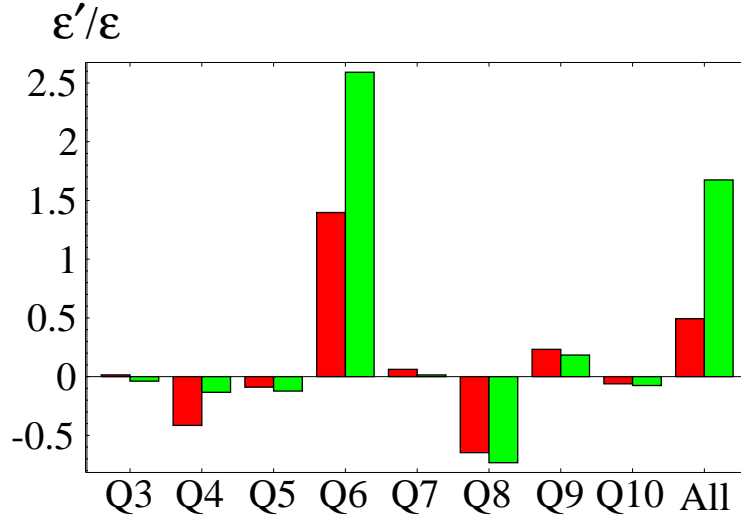


Figure 3: Predicting  $\varepsilon'/\varepsilon$ : a (Penguin) Comparative Anatomy of the München (dark gray) and Trieste (light gray) results in Fig. 1 (in units of  $10^{-3}$ ).

the  $X$ -bosons are computed within the ENJL model which improves the high-energy behavior. The  $\Delta I = 1/2$  rule is reproduced and the computed amplitudes show a satisfactory renormalization scale and scheme stability. A sizeable enhancement of the  $Q_6$  matrix element is found which brings the central value of  $\varepsilon'/\varepsilon$  at the level of  $3 \times 10^{-3}$ .

Other attempts to reproduce the measured  $\varepsilon'/\varepsilon$  using the linear  $\sigma$ -model, which include the effect of a scalar resonance with  $m_\sigma \simeq 900$  MeV, obtain the needed enhancement of  $\langle Q_6 \rangle$  <sup>34)</sup>. However, it is not possible to reproduce simultaneously the experimental values of  $\varepsilon'/\varepsilon$  and of the CP conserving  $K \rightarrow \pi\pi$  amplitudes.

Studies on the matching between long- and short- distances in large  $N$  QCD, with the calculation of the  $Q_7$  penguin matrix element and of  $\hat{B}_K$  at the NLO in the  $1/N$  expansion have been presented in ref. <sup>35)</sup>. However, a complete calculation of the  $K \rightarrow \pi\pi$  matrix elements relevant to  $\varepsilon'/\varepsilon$  is not available yet.

Without entering into the details of the various calculations I wish to illustrate with a simple exercise the crucial role of final-state interactions (in general of non-factorizable contributions) for  $\varepsilon'/\varepsilon$  and the  $\Delta I = 1/2$  selection rule. In order to do that I focus on two semi-phenomenological approaches.

A commonly used way of comparing the estimates of hadronic matrix elements in different approaches is via the so-called  $B$  factors which represent the ratio of the model matrix elements to the corresponding VSA values. However, care must be taken in the comparison of different models due to the scale dependence of the  $B$ 's and the values used by different groups for the parameters that enter the

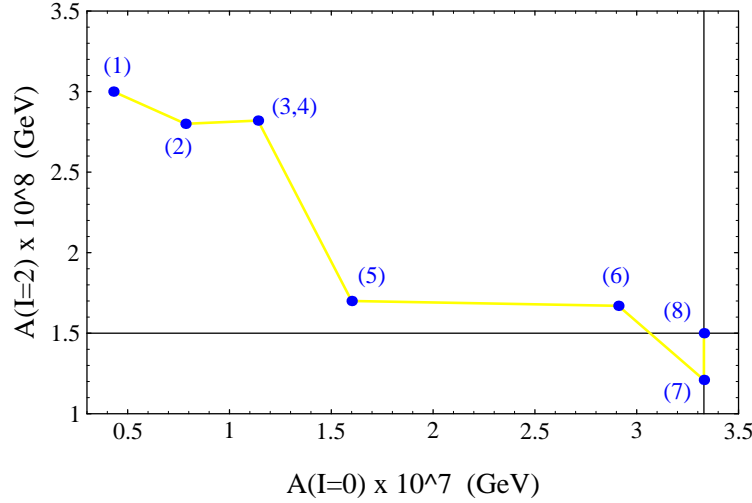


Figure 4: Anatomy of the  $\Delta I = 1/2$  rule in the  $\chi$ QM. See the text for explanations. The cross-hairs indicate the experimental point.

VSA expressions. An alternative pictorial and synthetic way of analyzing different outcomes for  $\varepsilon'/\varepsilon$  is shown in Fig. 3, where a “comparative anatomy” of the old Trieste and München predictions is presented.

From the inspection of the various contributions it is apparent that the different outcome on the central value of  $\varepsilon'/\varepsilon$  is almost entirely due to the difference in the  $Q_6$  component.

In the München approach <sup>17)</sup> the  $\Delta I = 1/2$  rule is used in order to determine phenomenologically the matrix elements of  $Q_{1,2}$  and, via operatorial relations, some of the matrix elements of the left-handed penguins. The approach does not allow for a phenomenological determination of the matrix elements of the penguin operators which are most relevant for  $\varepsilon'/\varepsilon$ , namely the gluonic penguin  $Q_6$  and the electroweak penguin  $Q_8$ . These matrix elements are taken around their leading  $1/N$  values (factorization).

In the semi-phenomenological approach of the Trieste group the size of the effects on the  $I = 0, 2$  amplitudes is controlled by the phenomenological embedding of the  $\Delta I = 1/2$  selection rule which determines the ranges of the model parameters: the constituent quark mass, the quark and the gluon condensates. In terms of these parameters all matrix elements are computed.

Fig. 4 shows an anatomy of the  $\chi$ QM contributions which lead to the experimental value of the  $\Delta I = 1/2$  selection rule for central values of the input parameters. Point (1) represents the result obtained by neglecting QCD and taking the factorized matrix element for the tree-level operator  $Q_2$ , which is the leading

electroweak contribution. The ratio  $A_0/A_2$  is thus found equal to  $\sqrt{2}$ : by far off the experimental point (8). Step (2) includes the effects of perturbative QCD renormalization on the operators  $Q_{1,2}$  <sup>36)</sup>. Step (3) shows the effect of including the gluonic penguin operators <sup>37)</sup>. Electroweak penguins <sup>38)</sup> are numerically negligible in the CP conserving amplitudes and are responsible for the very small shift in the  $A_2$  direction. Therefore, perturbative QCD and factorization lead us from (1) to (4).

Non-factorizable gluon-condensate corrections, a crucial model dependent effect entering at the leading order in the chiral expansion, produce a substantial reduction of the  $A_2$  amplitude (5), as it was first observed by Pich and de Rafael <sup>39)</sup>. Moving the analysis to  $O(p^4)$ , the chiral loop corrections, computed on the LO chiral lagrangian via dimensional regularization and minimal subtraction, lead us from (5) to (6), while the finite parts of the NLO counterterms calculated in the  $\chi$ QM approach lead to the point (7). Finally, step (8) represents the inclusion of  $\pi$ - $\eta$ - $\eta'$  isospin breaking effects <sup>40)</sup>.

This model dependent anatomy shows the relevance of non-factorizable contributions and higher-order chiral corrections. The suggestion that chiral dynamics may be relevant to the understanding of the  $\Delta I = 1/2$  selection rule goes back to the work of Bardeen, Buras and Gerard <sup>18)</sup> in the  $1/N$  framework with a cutoff regularization. A pattern similar to that shown in Fig. 4 for the chiral loop corrections to  $A_0$  and  $A_2$  was previously obtained in a NLO chiral lagrangian analysis, using dimensional regularization, by Missimer, Kambor and Wyler <sup>41)</sup>. The Trieste group has extended their calculation to include the NLO contributions to the electroweak penguin matrix elements <sup>42, 28)</sup>.

Fig. 5 shows the contributions to  $\varepsilon'/\varepsilon$  of the various penguin operators, providing us with a finer anatomy of the NLO chiral corrections. It is clear that chiral-loop dynamics plays a subleading role in the electroweak penguin sector ( $Q_{8-10}$ ) while enhancing by 60% the gluonic penguin ( $I = 0$ ) matrix elements. The NLO enhancement of the  $Q_6$  matrix element is what drives  $\varepsilon'/\varepsilon$  in the  $\chi$ QM to the  $10^{-3}$  ballpark.

As a consequence, the  $\chi$ QM analysis shows that the same dynamics that is relevant to the reproduction of the CP conserving  $A_0$  amplitude (Fig. 4) is at work in the CP violating sector, albeit with a reduced strenght.

In order to ascertain whether these model features represent real QCD effects we must wait for future improvements in lattice calculations <sup>9, 32)</sup>. On the other hand, indications for such a dynamics follow also from the analitic properties of the  $K \rightarrow \pi\pi$  amplitudes, as the dispersive analyses show <sup>6)</sup>. It is important to notice however that the size of the effect so derived is generally not enough to fully

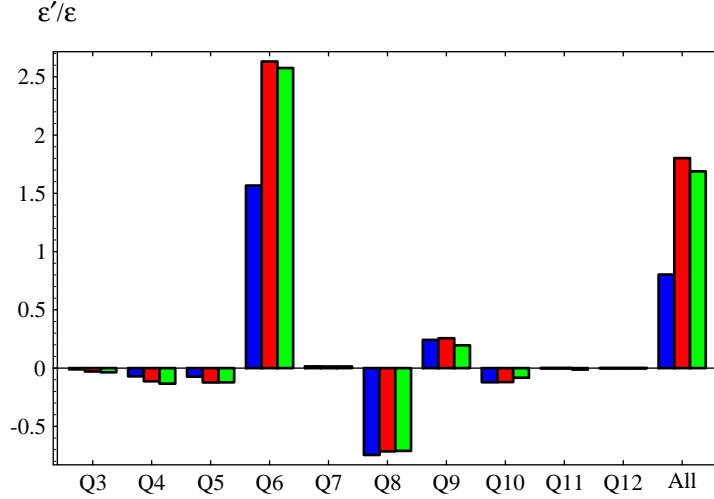


Figure 5: Anatomy of  $\varepsilon'/\varepsilon$  (in units of  $10^{-3}$ ) in the  $\chi$ QM approach. In black the LO results (which includes the non-factorizable gluonic corrections), in half-tone the effect of the inclusion of chiral-loop corrections and in light gray the complete  $O(p^4)$  estimate.

account for the  $\Delta I = 1/2$  rule. Other non-factorizable contributions are needed to further enhance the CP conserving  $I = 0$  amplitude, and to reduce the large  $I = 2$  amplitude obtained from perturbative QCD and factorization. In the  $\chi$ QM approach, for instance, the fit of the  $\Delta I = 1/2$  rule is due to the interplay of FSI (at the NLO) and non-factorizable soft gluonic corrections (at LO in the chiral expansion).

The idea of a connection between the  $\Delta I = 1/2$  selection rule and  $\varepsilon'/\varepsilon$  goes back a long way <sup>43)</sup>, although at the GeV scale, where we can trust perturbative QCD, penguins are far from providing the dominant contribution to the CP conserving amplitudes.

In summary, those semi-phenomenological approaches which reproduce the  $\Delta I = 1/2$  selection rule in  $K \rightarrow \pi\pi$  decays, generally agree in the pattern and size of the  $I = 2$  hadronic matrix elements with the existing lattice calculations. On the other hand, the  $\Delta I = 1/2$  rule forces upon us large deviations from the naive VSA for the  $I = 0$  amplitudes:  $B$ -factors of  $O(10)$  are required for  $\langle Q_{1,2} \rangle_0$ . Here is where lattice calculations suffer from large systematic uncertainties.

In the Trieste and Dortmund calculations, which reproduce the CP conserving  $K \rightarrow \pi\pi$  amplitudes, non-factorizable effects (mainly final-state interactions) enhance the hadronic matrix elements of the gluonic penguins, and give  $B_6/B_8^{(2)} \approx 2$ . Similar indications stem from recent  $1/N$  <sup>23)</sup> and dispersive <sup>6)</sup> approaches. The

direct calculation of  $K \rightarrow \pi\pi$  amplitudes and unquenching are needed in the lattice calculations in order to account for final state interactions. Further progress in this direction is awaited.

## References

1. A. Alavi-Harati *et al.*, *Phys. Rev. Lett.* **83**, 22 (1999); A. Barker, these Proceedings.
2. V. Fanti *et al.*, *Phys. Lett. B* **465**, 335 (1999); R. Granier de Cassagnac, these Proceedings.
3. J.H. Christenson *et al.*, *Phys. Rev. Lett.* **13**, 138 (1964).
4. S. Bertolini, J.O. Eeg and M. Fabbrichesi, *Rev. Mod. Phys.* **72**, 65 (2000).
5. E. Fermi, *Suppl. Nuovo Cim.* **2**, 17 (1955).
6. E. Pallante and A. Pich, *Phys. Rev. Lett.* **84**, 2568 (2000); J.F. Donoghue and E. Golowich, *Phys. Lett. B* **478**, 172 (2000); E. A. Paschos, hep-ph/9912230.
7. N.I. Mushkelishvili, *Singular Integral Equations* (Noordhoff, Gronigen, 1953), p. 204; R. Omnès, *Nuovo Cim.* **8**, 316 (1958).
8. A.J. Buras *et al.*, hep-ph/0002116.
9. M. Ciuchini, these Proceedings, hep-ph/0006056.
10. M. Gell-Mann and A. Pais, *Proc. Glasgow Conf.*, 342 (1955).
11. J. Gasser and U.G. Meissner, *Phys. Lett. B* **258**, 219 (1991); E. Chell and M.G. Olsson, *Phys. Rev. D* **48**, 4076 (1993).
12. S. Gardner and G. Valencia, *Phys. Lett. B* **466**, 355 (1999).
13. V. Cirigliano, J.F. Donoghue and E. Golowich, *Phys. Rev. D* **61**, 093001 (2000), *Phys. Rev. D* **61**, 093002 (2000); C.E. Wolfe and K. Maltman, *Phys. Lett. B* **482**, 77 (2000); S. Gardner and G. Valencia, hep-ph/0006240.
14. M. Serra, M. Nakao, D. Lucchesi, these Proceedings.
15. D. Bryman, these Proceedings.
16. H.Y. Cheng, hep-ph/9906403; hep-ph/9911202.

17. A.J. Buras, Proceedings of the Kaon 99 Conference, hep-ph/9908395.
18. W.A. Bardeen, A.J. Buras and J.M. Gerard, *Nucl. Phys. B* **293**, 787 (1987).
19. T. Hambye *et al.*, *Phys. Rev. D* **58**, 014017 (1998); hep-ph/9906434.
20. Y. Nambu and G. Jona-Lasinio, *Phys. Rev.* **122**, 345 (1961); J. Bijnens, *Phys. Rept.* **265**, 369 (1996).
21. A.A. Bel'kov *et al.*, hep-ph/9907335;
22. J. Bijnens and J. Prades, *JHEP* **9901**, 023 (1999).
23. J. Bijnens and J. Prades, hep-ph/0005189.
24. W. A. Bardeen, J. Bijnens and J.-M. Gérard, *Phys. Rev. Lett.* **62**, 1343 (1989); J. Bijnens, J.-M. Gérard and G. Klein, *Phys. Lett. B* **257**, 191 (1991); J. P. Fatelo and J.-M. Gérard, *Phys. Lett. B* **347**, 136 (1995).
25. J. Bijnens and J. Prades, *Phys. Lett. B* **342**, 331 (1995); *Nucl. Phys. B* **444**, 523 (1995); *JHEP* **9901**, 023 (1999); *JHEP* **0001**, 002 (2000).
26. S. Weinberg, *Physica A* **96**, 327 (1979); A. Manhoar and H. Georgi, *Nucl. Phys. B* **234**, 189 (1984).
27. S. Bertolini *et al.*, *Nucl. Phys. B* **514**, 63 (1998).
28. S. Bertolini *et al.*, *Nucl. Phys. B* **514**, 93 (1998).
29. S. Bertolini *et al.*, hep-ph/0002234.
30. M. Fabbrichesi, hep-ph/0002235.
31. D. Kaplan, *Phys. Lett. B* **288**, 342 (1992).
32. A. Soni, Proceedings of the  $B_{CP}$  99 Conference, hep-ph/0003092.
33. G. Martinelli, Proceedings of the Kaon 99 Conference, hep-ph/9910237.
34. Y.Y. Keum, U. Nierste and A.I. Sanda, *Phys. Lett. B* **457**, 157 (1999); M. Harada *et al.*, hep-ph/9910201; J.C.R. Bloch *et al.*, nucl-th/9910029.
35. S. Peris, M. Perrottet and E. de Rafael, *JHEP* **9805**, 011 (1998); M. Knecht, S. Peris and E. de Rafael, *Phys. Lett. B* **443**, 255 (1998), *Phys. Lett. B* **457**, 227 (1999); S. Peris and E. de Rafael, hep-ph/0006146.

36. M.K. Gaillard and B. Lee, *Phys. Rev. Lett.* **33**, 108 (1974); G. Altarelli and L. Maiani, *Phys. Lett. B* **52**, 413 (1974).
37. M.A. Shifman, A.I. Vainshtein and V.I. Zacharov, *JETP Lett.* **22**, 55 (1975); *Nucl. Phys. B* **120**, 316 (1977); *JETP* **45**, 670 (1977); J. Ellis *et al.*, *Nucl. Phys. B* **131**, 285 (1977), (E) **132**, 541 (1978); F.J. Gilman and M.B. Wise, *Phys. Lett. B* **83**, 83 (1979); *Phys. Rev. D* **20**, 2392 (1979); R.S. Chivukula, J.M. Flynn and H. Georgi, *Phys. Lett. B* **171**, 453 (1986).
38. J. Bijnens and M.B. Wise, *Phys. Lett. B* **137**, 245 (1984); A.J. Buras and J.M. Gerard, *Phys. Lett. B* **192**, 156 (1987); M. Lusignoli, *Nucl. Phys. B* **325**, 33 (1989); J.M. Flynn and L. Randall, *Phys. Lett. B* **224**, 221 (1989).
39. A. Pich and E. de Rafael, *Nucl. Phys. B* **358**, 311 (1991).
40. B. Holstein, *Phys. Rev. D* **20**, 1187 (1979); J.F. Donoghue, E. Golowich, B.R. Holstein and J. Trampetic, *Phys. Lett. B* **179**, 361 (1986); A.J. Buras and J.M. Gerard, in ref. 38).
41. J. Kambor, J. Missimer and D. Wyler, *Nucl. Phys. B* **346**, 17 (1990); *Phys. Lett. B* **261**, 496 (1991).
42. V. Antonelli *et al.*, *Nucl. Phys. B* **469**, 143 (1996).
43. M.A. Shifman, A.I. Vainshtein and V.I. Zacharov, in ref. 37); F.J. Gilman and M.B. Wise, in ref. 37).